

Chaos and Fractal Applications, by Marie McCrary

Grade Level: 8

Time Required: 1 class period

Goals & Objectives:

Goals: The goal of this lesson is to show students how chaos works numerically using a graphing calculator.

Objectives:

Students will be able to:

1. Practice using a graphing calculator.
2. Understand the meaning of parameters, iterations, and initial conditions.
3. Understand how changing the parameters and initial conditions affect the solutions.
4. Understand the difference between steady state, oscillating, and chaotic solutions.
5. Demonstrate and understand the fundamental property of chaos, called sensitive dependence on initial conditions.
6. Understand that fractals can be used as a mathematical representation of processes that occur in nature, such as ferns and other plants.

Materials:

For each group of students

1. One TI graphing calculator per student

NJCCCS Addressed:

4.1.A.6, 4.1.A.7, 4.1.B.1, 4.1.B.5, 4.1.C, 4.1.D, 4.3.A, 4.3.B, 4.3.C, 4.3.D.1, 4.3.D.2, 4.3.D.4, 4.5.A, 4.5.B, 4.5.C, 4.5.D, 4.5.E

Procedure:

- Give the students an introduction to chaos, show fractal PowerPoint slides, and review concepts such as:
 1. Parameter – the parameter r defines certain behaviors of the solution
 2. Initial condition – the initial value for the input x
 3. Iteration – the process of substituting the solution as the new variable
- Introduce the logistic map to the class as $\text{answer} = r * x - x * x$. In order to iterate the solutions in a graphing calculator, have the students follow the following algorithm:
 1. Input the initial condition, x_0 and press enter.
 2. For any value of r , input $r * \text{ans} - \text{ans} * \text{ans}$ and press enter. The solution is the new answer, x_1 .
 3. Once x_1 is recorded, the student can easily iterate the equation by pressing enter. The solution is automatically substituted for x during each iteration.
- Different behaviors arise for different values of r . Have the students iterate the logistic map for the following values of r :

1. $r = 1/2$: Students should see that the solutions will tend toward 0. Demonstrate that even though the calculator will eventually read zero, the solution never gets to exactly zero and this is an approximation error. This is known as a die-out.
2. $r = 2$: The solution will stabilize to $(r - 1)/r$. In this case, the solution will stabilize to $1/2$. This is known as a steady state or stable solution.
3. $r = 3$: The solution will oscillate between two values dependent on r .
4. $r = 4$: The solutions are chaotic. Have the students input different initial conditions and record their answers. They should see that even for slightly different initial conditions (for example, $x_0 = 2.1$ and $x_0=2.12$), the solutions will eventually vary by a much greater degree. This is known as sensitive dependence on initial conditions and is one of the fundamental properties of chaos.

Accommodations:

- Students are placed in groups of mixed ability and assigned specific tasks
- Images and written guidelines are provided for visual learners and English language learners

Assessment:

- Make observations as students work together
- Have students record the initial conditions and solutions obtained to be collected
- Allow students to discuss their findings to the class.

Follow-up/Homework:

For homework, students can investigate

- practical applications of chaos
- the history of the development of chaos theory
- other types of chaotic maps
- other examples of where fractals appear in nature