

How Many Combinations Are There?

Original lesson to be used in conjunction with the “Substances Investigation” from the Foss Kit

Topic: Combinations and Permutations

Grade: 8th grade

Time Allotted: 1 – 2 class periods (depending on the level of the students)

Prior Knowledge: Before this lesson, students should have started the Substances Investigation in the Foss Kit. This lesson should be used prior to the start of students testing different combinations of substances to determine the mystery mix. This lesson can also be done as a stand – alone lesson.

- Edges
- Vertices
- Graphs

Materials:

- TI – 73 calculators
- ~ 50 pieces of string approximately 3 feet long
- Worksheet
- Pencil / colored pencils

Goals:

- Determine the possible number of combinations to test for the mystery mixture investigation
- Calculate the number of permutations in which order IS important
- Calculate the number of permutations in which order IS NOT important
- Calculate the number of any given permutations by listing the combinations, using graph theory to count the number of combinations
- Use TI – 73 to calculate number of permutations using binomial coefficients
- **Key definitions:** Combinatorics, permutations, ordering, discrete, factorials

Objectives: Students should be able to...

- Understand that mathematical results can be calculated using different techniques, all of which are valid
- Understand how doing a small scale experiment can lead to general results
- Understand how math is used to determine the next step in a scientific process

NJCCCS:

- 5.1.A2, 5.1.B1, 5.3.C1, 5.3.D1

Procedures:

- Have students sit in groups of three
- **Define: Permutations** – rearranging in an ordered fashion objects or values. You can either rearrange ALL the objects in the group or you can rearrange and group a particular number of objects from the entire group (like in the Substances investigation). Combinatorics is an area of math in which you count discrete structures.
- **In your groups of three, I want you to stand in a straight line front – to – back. For this first trial, we are going to say that order is not important. This means that who is front and behind you does not matter, all that matters is you can't repeat the same exact line which would mean that you are all in the same spot. How many combinations will you have? Write the possible combinations on your worksheet. The answer is 6. It is $3!$ since either of three students can be first in line, which means that two of the students can then be second in line, and only one student will be third in line.**
- Give groups 10 pieces of string. Ten is just a nice number, but you can give them any amount of pieces of string greater than 6. Avoid giving groups six pieces of string so they do not simply guess the answer.
- **If we wanted to count the same thing, number of combinations of three students, how could we do this by creating a graph? Students are the vertices and the pieces of string are the edges. Try it! Students should come up with the same result. This is not a coincidence, you are counting exact the same thing but in a different way.**
- **What if I wanted to know the possible combinations of all students in this class? Take $n!$ where n is the number of students in that particular class**

- **Now that you can count the number of possible combinations using the whole group, let's change the game a little bit.**
- Depending on time constraints you can either have the students form larger groups at this point or keep them in their groups of three.
- **What I am going to ask you to do now is to repeat the investigation you just completed, however, now I want to know all the possible combinations that consist of ONLY TWO STUDENTS.** If students are confused give them a hint by using Ms. Zayat, Ms. Alapa, and Mrs. Cheteyan. We are a group of three but if I pair Mrs. Cheteyan and Ms. Alapa together then that is one group of two. The students' job is to figure out how many combinations there are.
- **STOP! Before you start, you need to consider one thing. Does order matter?** *Yes. A group with Mrs. Cheteyan and Ms. Alapa is the SAME group as Ms. Alapa and Mrs. Cheteyan. Think about the substances investigation, does it matter which substance I include first? Isn't mixing table salt and road salt the same as mixing road salt and table salt? Yes.*
- **Now you may begin. First start by listing the possible combinations, then try to graph the number of combinations. Do you think the number will be the same? Why?**
- Give the students all but five minutes to finish this investigation. When they are done, have them share their answers. If they worked in groups of three their answer will be 3. If they worked in groups of six their answer will be 15.
- **In the graph, the results should have been exactly the same. Draw the result so groups can check their answers.**
- **Finally, show them the "n choose k" formula button, n is the number of students in the group, k is the number you want to choose. Have each student check the results they just obtained.**
- **Wrap Up:** *Now that you know how to use the calculator to solve n choose k problems. How many substances will you have to test to determine the mystery mix? $n = 9$ $k = 2$, answer: 36*

Accommodations:

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Any and all worksheets or data sheets come at this point. If you want to add homework add it at the end of procedures, there is no need for an assessment section.

4. Draw the graphical representation of question 3?

5. What is the difference between combinations and permutations? What is the same?

6. Using the knowledge you've gained thus far, how many possible mixtures could the mystery mixture be? (Assume we have 9 substances to choose from.)